

8.5: Properties of Logarithms (pg 509)

Ex #1: Use $\log_4 3 \approx 0.7925$

$$\begin{aligned} &\log_4 192 \\ &\log_4 4^3 \cdot 3 \\ &3 \log_4 4 + \log_4 3 \\ &3 \log_4 4 + \log_4 3 \\ &3(1) + 0.7925 \\ &\underline{3.7925} \end{aligned}$$

$$\begin{aligned} &192 \\ &4 \uparrow 48 \\ &4 \uparrow 12 \\ &4 \uparrow 3 \end{aligned}$$

$$\begin{aligned} \log_b m^p &= p \log_b m \\ \log_x ab &= \log_x a + \log_x b \\ \log_x \frac{a}{b} &= \log_x a - \log_x b \end{aligned}$$

1A) Use $\log_4 2 = .5$

$$\begin{aligned} &\log_4 32 \\ &\log_4 2^5 \\ &5 \log_4 2 \\ &5(.5) = \underline{2.5} \end{aligned}$$

Ex 2: $pH = \log_{10} \left[\frac{1}{H^+} \right]$ $pH = 4.2$

$$\begin{aligned} 4.2 &= \log_{10} \frac{1}{H^+} \\ 4.2 &= \log_{10} 1 - \log_{10} H^+ \\ 4.2 &= \log_{10} 1 - \log_{10} H^+ \\ -1 & \quad \downarrow \\ -4.2 &= \log_{10} H^+ \\ \log_{10} H^+ &= -4.2 \\ \underline{10^{-4.2} = H^+} \end{aligned}$$

2A) $L = 10 \log_{10} R$

$$\begin{aligned} \frac{L}{10} &= \log_{10} R \\ \log_{10} R &= \frac{L}{10} \\ 10^{\frac{L}{10}} &= R \end{aligned}$$

Ex #3) $\log_2 25 = 2.3219$

$$\begin{aligned} &\log_2 25 \\ &\log_2 5^2 \\ &2 \log_2 5 \\ &2(2.3219) = \underline{4.6438} \end{aligned}$$

3A) $\log_3 7 = 1.7712$

$$\begin{aligned} &\log_3 49 \\ &\log_3 7^2 \\ &2 \log_3 7 \\ &2(1.7712) = \underline{3.5424} \end{aligned}$$

Ex #4) $\log_6 1 + \log_6 (x-9) = 2$

$$\begin{aligned} \log_6 x(x-9) &= 2 \\ (x-12)(x+3) &= 0 \\ x-12=0 \quad x+3=0 \\ +12-12 \quad -3-3 \\ \underline{x=12} \quad \underline{x=-3} \end{aligned}$$

$$\begin{aligned} x^2 &= x^2 - 9x \\ -36 &= x^2 - 9x \\ -36 & \quad -36 \\ \underline{0} &= x^2 - 9x - 36 \\ x^2 & \quad -12+3 \\ (x-12)(x+3) &= 0 \end{aligned}$$

$$\begin{aligned} \log_6 12 + \log_6 (12-9) &= 2 \\ \log_6 12 + \log_6 3 &= 2 \\ \log_6 6^2 &= 2 \\ 2 \log_6 6 &= 2 \\ 2(1) &= 2 \end{aligned}$$

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1A) Use $\log_4 2 = .5$

$$\begin{aligned} &\log_4 32 \\ &\log_4 2^5 \\ &5 \log_4 2 \\ &5(.5) = \boxed{2.5} \end{aligned}$$

Ex 2: pH = $\log_{10} \frac{1}{H^+}$ pH = 4.2

$$\begin{aligned} 4.2 &= \log_{10} \frac{1}{H^+} \\ 4.2 &= \log_{10} 1 - \log_{10} H^+ \\ \frac{4.2}{-1} &= \frac{-\log_{10} H^+}{-1} \end{aligned}$$

$$\begin{aligned} -4.2 &= \log_{10} H^+ \\ \log_{10} H^+ &= -4.2 \end{aligned}$$

$$\boxed{10^{-4.2} = H^+}$$

2A) $L = 10 \log_{10} R$
 $\frac{60}{10} = \frac{10 \log_{10} R}{10}$
 $6 \log_{10} R = \log_{10} R$
 $\log_{10} R = 10^{5.6}$
 $10^6 = R$

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Ex #4) $\log_6 12 + \log_6 (x-9) = 2$

$$\log_6 x(x-9) = 2 \quad \log_6 12 + \log_6 (12-9) = 2$$

$$\begin{aligned} (x-12)(x+3) &= 0 \\ x-12=0 & \quad x+3=0 \\ +12-12 & \quad -3-3 \\ \hline \boxed{x=12} & \quad \cancel{x=-3} \end{aligned}$$

$$\begin{aligned} 6^2 &= x^2 - 9x \\ 36 &= x^2 - 9x \\ -36 & \quad -36 \\ \hline 0 &= x^2 - 9x - 36 \\ & \quad \quad \quad -12 \cdot 3 \\ & (x-12)(x+3) = 0 \end{aligned}$$

$$\begin{aligned} &\log_6 12 \cdot 3 = 2 \\ &\log_6 6^2 = 2 \\ &2 \log_6 6 = 2 \\ &2(1) = 2 \end{aligned}$$